ALGEBRA Ecosystem: Decentralized exchange

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Abstract. In this article, we review main features of the Algebra Decentralized Exchange (DEX) and compare it with other DEXs and their approach to decentralized trading. The key differences between Algebra and other exchanges are dynamic fees, built-in farming, and deflationary tokens support. We are taking a closer look at these features and exploring exchange users' benefits.

1. Dynamic Fees

Introduction

Decentralized exchanges (DEX) are a fundamental constituent of the rapidly-developing field of decentralized finance. The functioning of the financial system is impossible without a stable and reliable ability to exchange assets. This is why a number of developers and researchers are focused on the development of exchanges that would enable this financial system. In addition to breakthrough technologies like Uniswap, there is work being done on a wide range of alternative approaches and solutions.

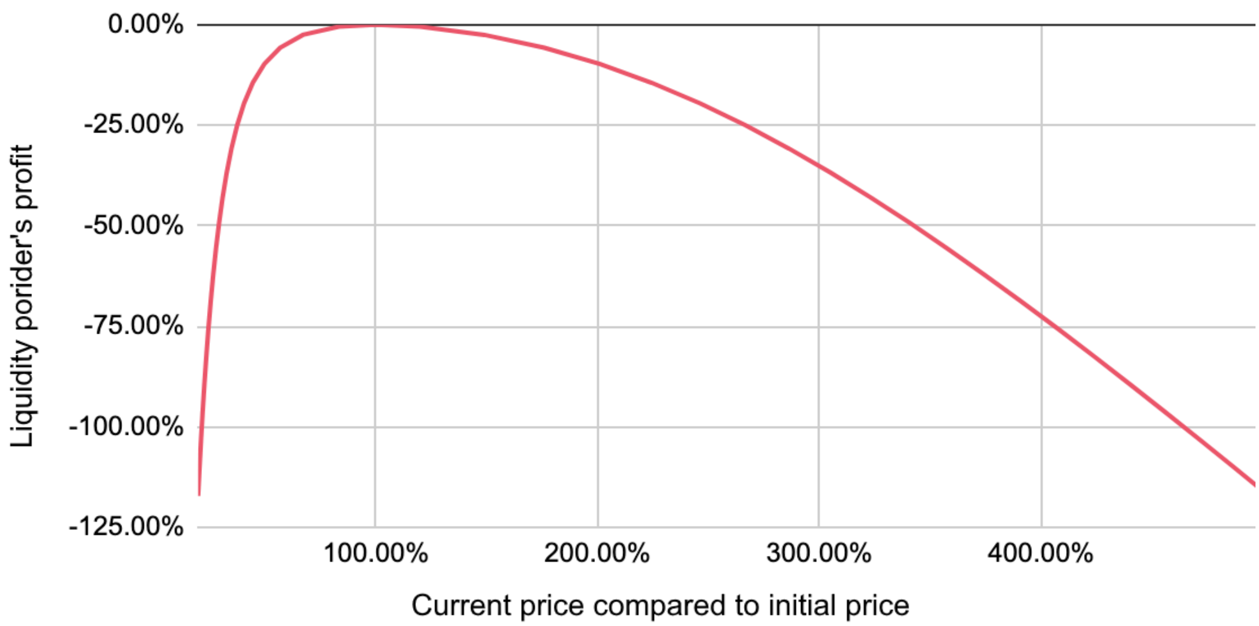
The leading direction in the field of DEXs (decentralized exchanges) is the use of automated market makers – AMM, which provide users with the opportunity to conduct instant exchanges without the participation of third parties as facilitators. The standard approach to implementing AMMs at the moment is to constantly follow some invariant, particularly the constant market function. For Uniswap V2 and other similar AMMs, it can be expressed as follows:

$$X \cdot Y = k \tag{1}$$

where X and Y, respectively, are the reserves of the first and second asset in the pool, and k is a constant determined when creating a pair. This formula provides a simple calculation of the swap results, the stability of the exchange and its functioning at any time. However, this system relies completely on providing a sufficient number of reserves to maintain an acceptable level of liquidity and reflect the real market price.

The role of providing reserves is assigned to liquidity providers who place their assets in exchange for a share of the fees collected during the trading. For this reason, the provision of attractive and economically reasonable conditions for liquidity providers is fundamental for the viability and functionality of an AMM. While examining the attractiveness of the liquidity provision strategy, it is rational to compare it with the "zero strategy" – storing assets on the account without performing any actions, it is also referred to as a “hold strategy”. The dependence of the liquidity provider's profit compared to the “hold strategy” when the price moves under zero fee, is depicted in Figure 1.

Fig. 1



Comparison of The Liquidity Provider's Profit Relative To The Hold Strategy

The phenomenon of a provider's negative profit when the price moves is called impermanent loss, since the losses do not occur when the price returns to the initial level. The nature of impermanent losses significantly depends on the type of constant function – when using an invariant other than (1), the depicted loss curve may be more flat or steep. Due to the impermanent loss, the volatile price of an asset is a risk factor for liquidity providers, which should be compensated by a high level of potential profit from fees.

The key innovations implemented for the first time in Uniswap V3 are aimed at solving the problems described above when providing liquidity. When using the technology of concentrated liquidity, assets are placed at specified price intervals, called positions (Fig. 2). This liquidity is used during trading and collects a fee only if the current price is within the range set by the position, which increases the efficiency of using the liquidity provider's funds. Using positions allows you to achieve an uneven distribution of liquidity and, as a result, different price behaviour and temporary losses.

Fig. 2 Examples of Liquidity Placement Ranges

Therefore, the placement of assets within price ranges not only increases the efficiency of funds but also makes the overall market function much more flexible due to the arbitrary distribution of liquidity. Due to this, liquidity providers have the ability to independently adjust the conditions that minimize the impermanent loss. Another important factor is the presence of several pools with different fees for each pair of assets. Via self-regulation, based on supply and demand, the ability to choose the applied pool theoretically leads to the establishment of optimal compromised conditions for all participants in the trading process.

However, the effectiveness of the overall liquidity is significantly reduced when several pools are used on a certain pair, all at once. For instance, only 72% of providers choose a pool with a 0.3% fee on the ETH/USDC pair, as can be seen in Figures 3.1 - 3.4. While the remaining 28% virtually exist separately. The less liquidity is provided for a specific period, the less profitable the exchange is due to a strong price change during the exchange. Consequently, such distribution of assets leads to the fact that in a pool with 0.3%, exchanges move the price more strongly than in a situation where all the liquidity is provided in one pool. "One pool" approach is used in Algebra.

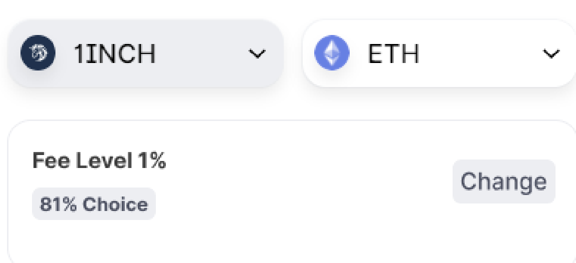


Figure 3.1 1INCH\ETH Pool

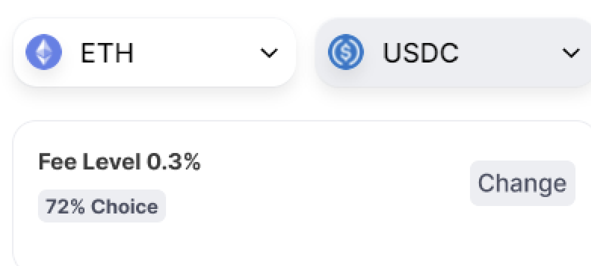
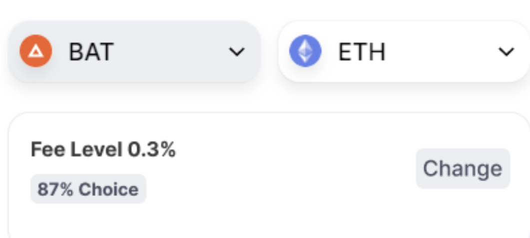


Figure 3.2 ETH\USDC Pool



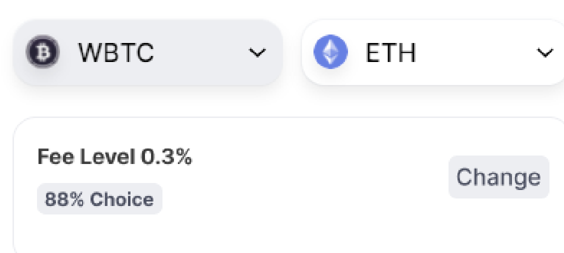


Figure 3.3 BAT\ETH Pool

Figure 3.4 WBTC\ETH Pool

Optimal Conditions

All participants interacting with a DEX can be divided into three categories:

1. Regular Traders
2. Arbitrageurs
3. Liquidity Providers

Each group of participants has its own goals and strives to reach maximum profit with minimal risks. The interests of groups can either coincide or contradict. On the one hand, liquidity providers are interested in maximizing the profit from their investments, which is positively influenced by high volume, high fees and low volatility. On the other hand, high fees contradict the interests of users who carry out trades.

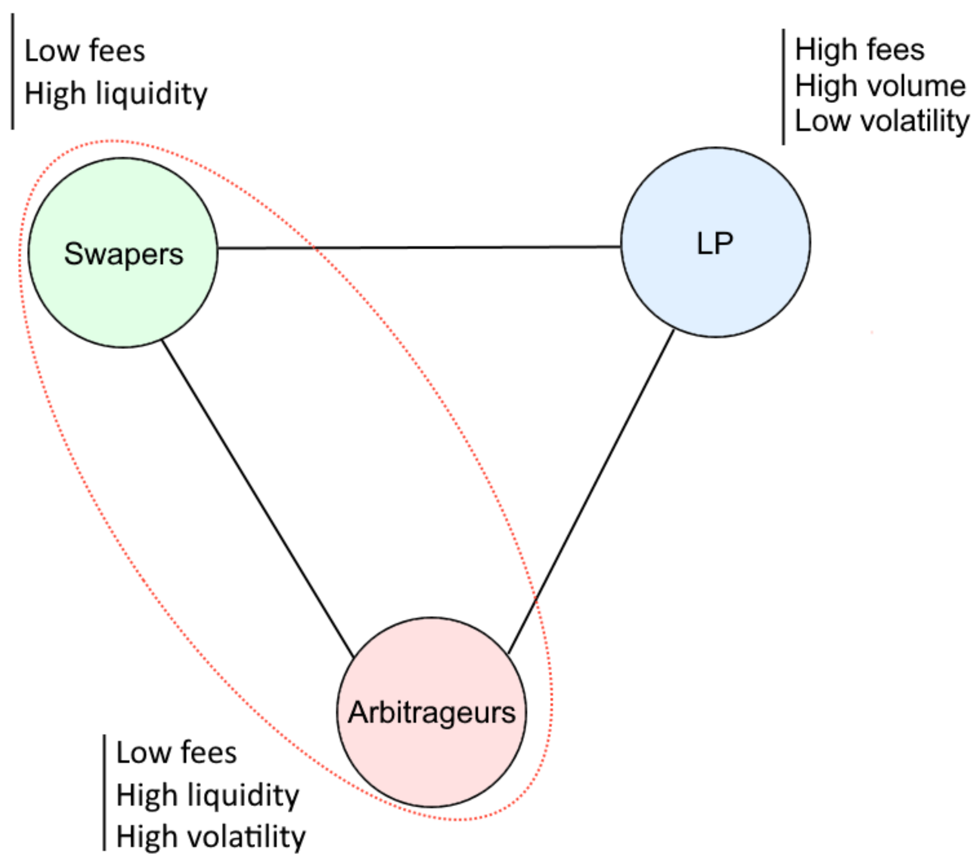


Fig. 4 User Groups and Their Interests

The effective functionality of a DEX requires a balanced and compromised solution that meets the interests of all parties. Since the entire system can be considered closed – money does not emerge from anywhere – an increase in the profit of one of the participants, usually, has the opposite effect on others.

As already mentioned before, the fee size is one of the tools for redistributing profits and, consequently, regulating the balance of interests. The presence of three pool options for each pair in Uniswap V3 is precisely aimed at maintaining a balance of interests – the options are initially designed to optimally match three different asset classes, such as stable pairs, normal and exotic (Fig. 5).

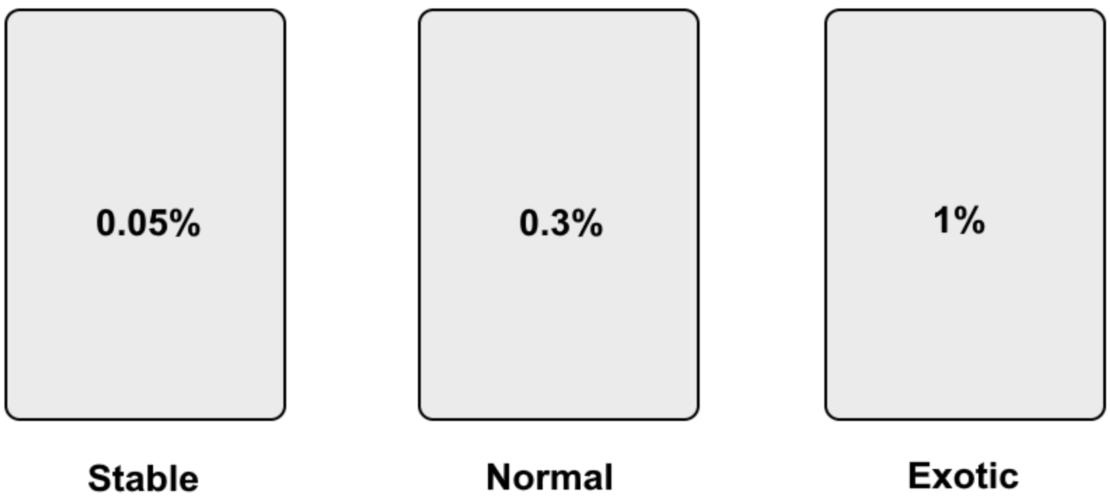


Fig. 5 Uniswap V3 Pools

However, as already mentioned, this approach has a number of disadvantages: the dispersion of liquidity, the complication of interaction with DEX, etc. In addition, the nature of the assets' behavior rarely remains the same for a long period of time. This fact can be demonstrated by the history of ETH prices to USDC (Fig. 6). During periods of high demand, volatility increases significantly, the price begins to change significantly and drastically – at this time, the losses of liquidity providers increase, which should force them to move assets to a pool with higher fees. On the other hand, there are also time intervals with a more stable price.

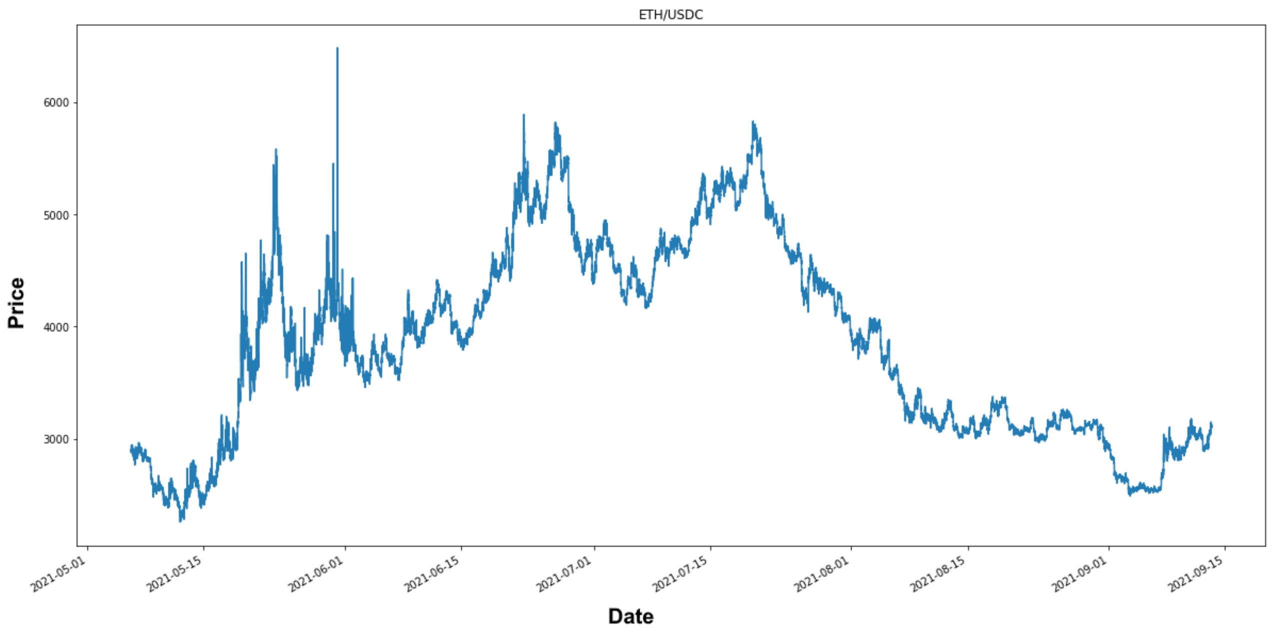


Fig. 6 History of ETH/USDC Price

However, the mechanism of the fees' correction, with the help of several pools, turns out to be very inertial in such situations – it is unclear what the further behavior of the asset will be, and also which pool will provide the greatest liquidity and trading volumes. This is the reason for the initially selected "type" of the asset and the pool to be mainly preserved, which leads to inferior conditions and lost profits for participants.

To solve this problem, Algebra offers a dynamic fee mechanism that will enable the concentration of liquidity in one pool and guarantee the balance between participants, increasing the income of each of the parties.

Dynamic Fee Mechanism

To find the optimal commission amount, depending on the nature of the asset's behavior, the following indicators are monitored:

1. Volatility
2. Liquidity
3. Trading Volume

High liquidity is supported by beneficial conditions for providers, which consist of high trading volume with low volatility. In this case, the following situations are possible for providers:

	High volume	**Low volume**
High volatility	Average income	Negative income
Low volatility	High income	Average income

Table 1 Dependence of Profitability For LP On Market Behavior

From this, the following conclusions can be drawn:

When the volatility is high, it is necessary to increase the fee to compensate potential losses of liquidity providers;

When the volume is low and there is sufficient liquidity, fees should be reduced in order to attract more volume.

This is why Algebra offers a complex formula for determining the current fees in the pool, taking into account changes in the parameters of volatility, trading volume and current liquidity.

Volatility

First of all, based on the time-weighted moving average (2), the current volatility is calculated in the form of a variance (3).

In this case:

τ - the time points corresponding to the available price observations,

$P(\tau)$ - the value of the price at the τ moment,

$\Delta t(\tau)$ - the time during which this price was maintained,

T- time period (in seconds). Volatility, average values of the volume and the liquidity for every time point are calculated during the period.

$$TWMA \quad \overline{P}(t) = \frac{1}{T} \sum_{\tau \in [t-T,\, t]} P(\tau) \cdot \Delta t(\tau)$$

$$volatility \quad \delta(t) = \frac{1}{T} \sum_{\tau \in [t-T,\, t]} \left(P(\tau) - \overline{P}(\tau)\right)^2$$

It should be noted that these statistical parameters are calculated for the logarithmic prices (ticks), which removes the dependence on the absolute value of the price – a price change from 1000 to 1100 will not cause greater volatility than a change from 100 to 110. So actual used formulas are different:

$$tick(P) = log_{1.0001}\sqrt{P}$$

$$\begin{aligned} TWMA \quad \overline{tick}(t) &= \frac{1}{T}\sum_{\tau\in[t-T,\,t]} tick(P(\tau))\cdot\Delta t(\tau) = \\ &= \frac{1}{T}\sum_{\tau\in[t-T,\,t]} log_{1.0001}\sqrt{P(\tau)}^{\Delta t(\tau)} = \\ &= log_{1.0001}\sqrt[T]{\prod_{\tau\in[t-T,\,t]}\sqrt{P(\tau)}^{\Delta t(\tau)}} = \\ &= log_{1.0001}\sqrt{\widehat{P}(t)} \end{aligned} \tag{2}$$

$$\begin{aligned} volatility \quad \delta(t) &= \frac{1}{T}\sum_{\tau\in[t-T,\,t]} \left(tick(P(\tau)) - \overline{tick}(t)\right)^2 = \\ &= \frac{1}{T}\sum_{\tau\in[t-T,\,t]} \left(tick(P(\tau)) - \overline{tick}(\tau)\right)^2 = \\ &= \frac{1}{T}\sum_{\tau\in[t-T,\,t]} \left(log_{1.0001}\sqrt{\frac{P(\tau)}{\widehat{P}(\tau)}}\right)^2 \end{aligned} \tag{3}$$

The received volatility value is then used to determine the base fee level. To do this, a formula based on the sum of two sigmoids is used. The standard sigmoid is given by the formula (4).

$$y = \frac{1}{1+e^{-x}} \tag{4}$$

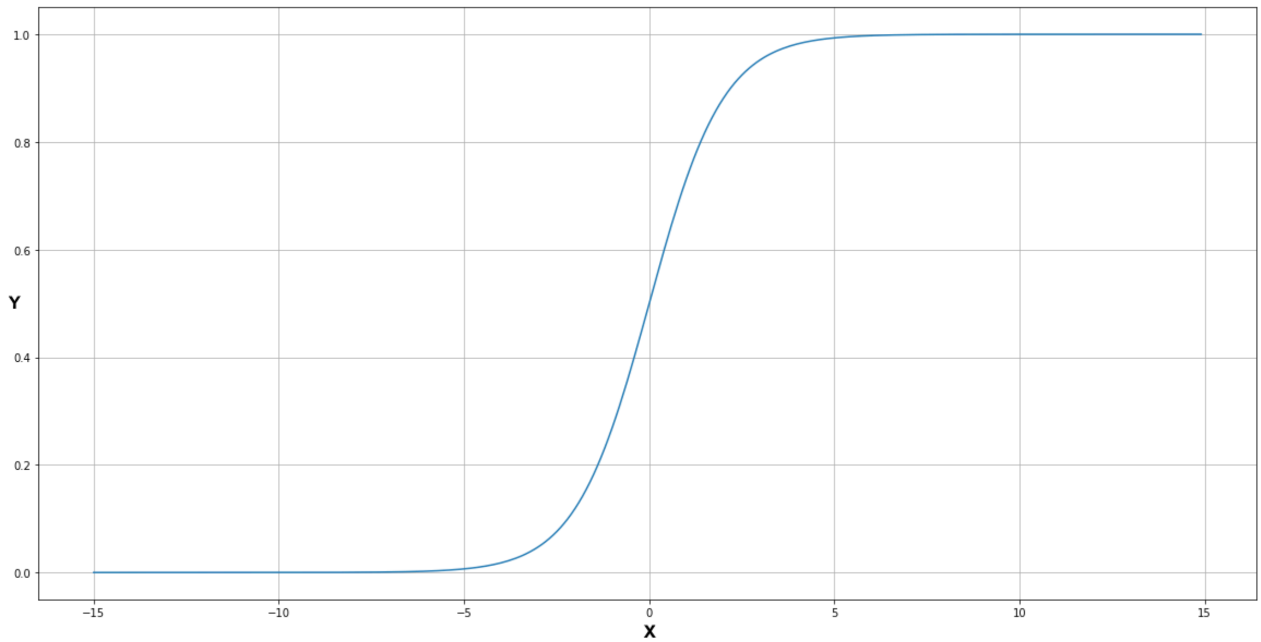


Fig. 7 Graph of The Standard Sigmoid Function

A small modification allows us to create the following formula, which provides considerably more flexibility:

$$y = \frac{\alpha}{1+e^{\gamma(\beta - x)}} \tag{5}$$

Technically, contracts represent gamma as a divisor, not a multiplier. So the exponent part looks like:

$$e^{(\beta - x)/\hat{\gamma}}$$

The following coefficients have been added to the formula:

- α - allows scaling the maximum value of the sigmoid
- β - is responsible for the linear shift along the x-axis
- γ - regulates the "steepness" of the sigmoid

In this case, you can derive the following formula:

$$y = \frac{\alpha_1}{1+e^{\gamma_1(\beta_1 - x)}} + \frac{\alpha_2}{1+e^{\gamma_2(\beta_2 - x)}} \tag{6}$$

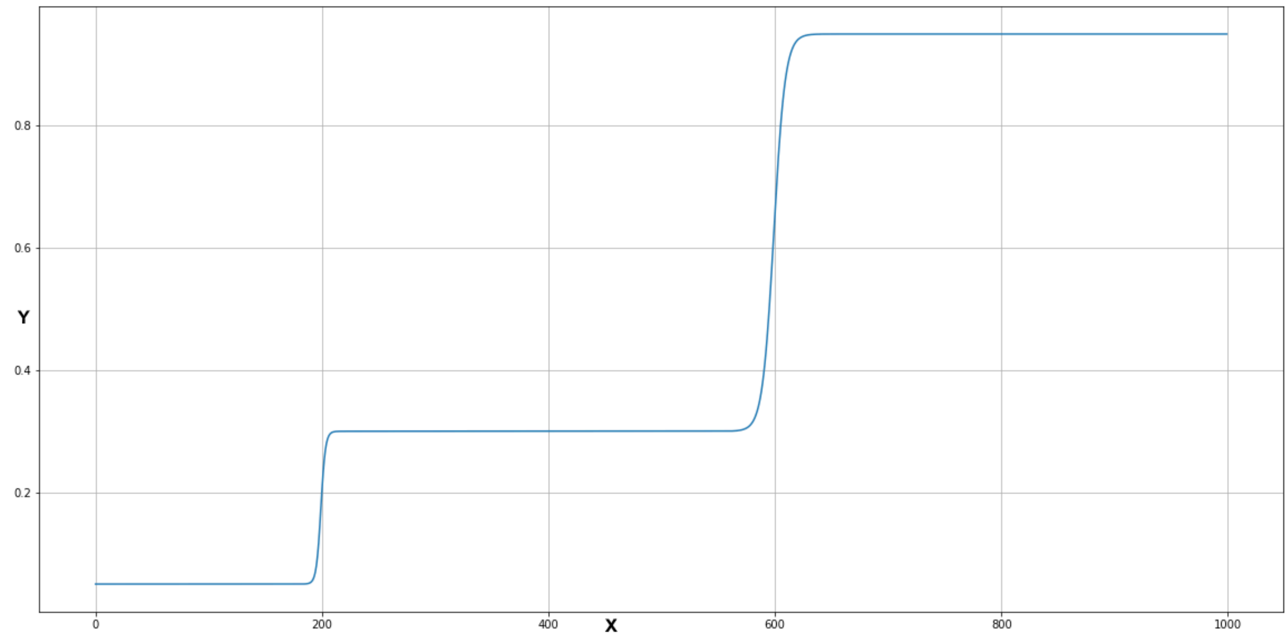
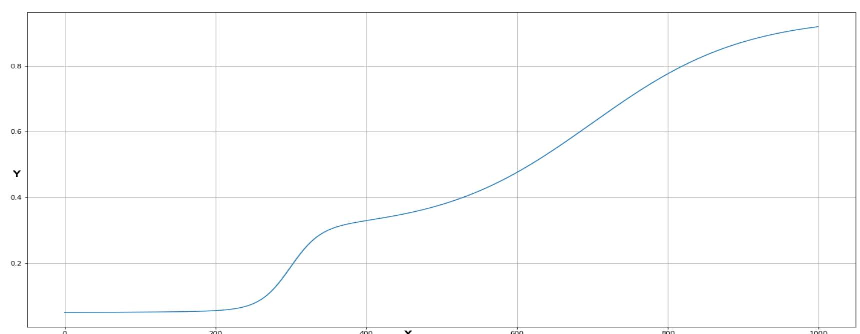
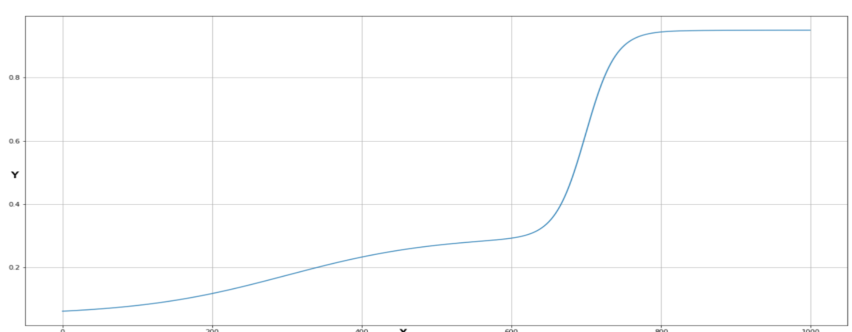
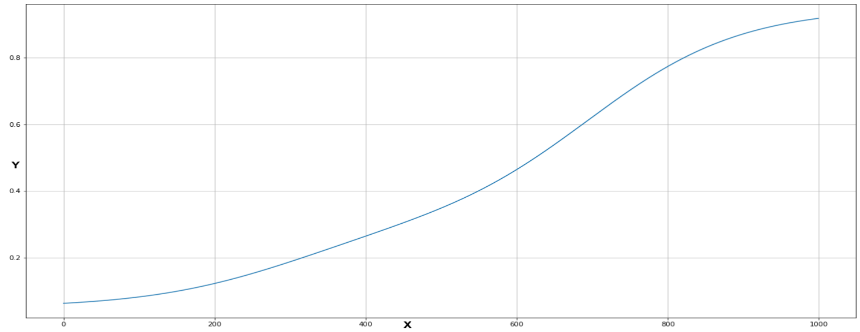


Fig. 8 Step Function of Sigmoids

By changing the coefficients, it is possible to provide a wide range of different options:







The derived formula is convenient for calculation and has sufficient flexibility, which allows you to set the dependence of the fees' base level on volatility:

$$Fee_{volatility}(t) = \frac{\alpha_1}{1+e^{\gamma_1\left(\beta_1-\delta(t)\right)}} + \frac{\alpha_2}{1+e^{\gamma_2\left(\beta_2-\delta(t)\right)}} \tag{7}$$

Formula (7) can be used to calculate the commission, but it does not take into account the other important factors – volume and liquidity.

Volatility resampling

Additionally, it should be noted that the update of information about the price does not occur every second. For this reason, it is necessary to resample records at a frequency of 1 record per second.

As we add each new entry, we know the following:

T - time elapsed since previous recording

$tick(T)$ - current tick value

$tick(0)$ - tick value at the moment of previous record

$\overline{tick}(T)$- current average tick value

$\overline{tick}(0)$ - average tick value at the moment of previous record

In this case, the change in tick and average tick over the past time can be linearly interpolated by two functions:

$$tick(t) \ = \ k\,t \ + \ b$$

$$\overline{tick}(t) \ = \ p\,t \ + \ q$$

Where *k, b, p, q* are corresponding constants:

$$k \ = \frac{1}{T}\,(tick(T) \ - tick(0))$$

$$p \ = \frac{1}{T}\,(\overline{tick}(T) \ - \overline{tick}(0))$$

$$b \ = tick(0)$$

$$q \ = \overline{tick}(0)$$

So formula (3) can be approximated over a time range using:

$$volatilityOnRange(t_0, \ t_1) \ = \frac{1}{t_1 - t_0} \sum_{t = t_0}^{t_1} (tick(t) \ - \ \overline{tick}(t))^2 =$$

$$= \ \frac{1}{t_1 - t_0} \sum_{t = t_0}^{t_1} ((k \ - \ p)\,t \ + \ (b \ - \ q))^2 \ =$$

$$= \frac{1}{t_1 - t_0} \sum_{t = t_0}^{t_1} ((k \ - \ p)^2\,t^2 \ + 2\,(b \ - \ q)(k \ - \ p)\ t \ + (b \ - \ q)^2) \ =$$

$$\frac{1}{t_1 - t_0}\,((k \ - \ p)^2 \sum_{t = t_0}^{t_1} t^2 \ + 2\,(b \ - \ q)(k \ - \ p) \ \sum_{t = t_0}^{t_1} t \ + (t_1 - t_0)(b \ - \ q)^2$$

This approximation is used in contracts to correctly increase the volatility accumulator.

Volume

It is known that the absolute values of the trading volume for different assets can differ significantly – $1,000,000 is not the same as 1,000,000 ETH. For this reason, it is necessary to build a general indicator that will uniformly characterize the intensity of trading for different assets.

In the case of the market function (1), the amount of liquidity is calculated using the formula:

$$L = \sqrt{X \cdot Y} \tag{8}$$

Knowing the trading volume of each of the assets in the pool, you can get a value that characterizes the amount of liquidity used:

$$\widetilde{L} = \sqrt{V_1 \cdot V_2} \tag{9}$$

where V_1 and V_2, respectively, are the trading volumes of assets 1 and 2. In this case, the following ratio characterizes the level of intensity of use of the available liquidity in the pool:

$$I_t = \frac{\widetilde{L}_t}{L_t} \tag{10}$$

By using the formula (5) and determining the necessary threshold values, it is possible to create a "regulator" that reflects the "sufficiency" of the trading volume at the moment:

$$\frac{\alpha_R}{1+e^{\gamma_R\left(\beta_R - I_t\right)}} \tag{11}$$

It is reasonable to choose the coefficient α_R equal to 1, to let the trading volume play the role of a threshold regulator that reduces the commission at low exchange activity.

The constant Fee_{base} is used as the minimum possible fee value.

Then the final fee function turns out to be as follows:

$$Fee(t) = Fee_{base} + Fee_{volatility}(t) \cdot \frac{1}{1+e^{\gamma_R(\beta_R - I_t)}} \tag{12}$$

More detailed:

$$Fee(t) = Fee_{base} + \\ + \left(\frac{\alpha_1}{1+e^{\gamma_1(\beta_1 - \delta(t))}} + \frac{\alpha_2}{1+e^{\gamma_2(\beta_2 - \delta(t))}} \right) \cdot \frac{1}{1+e^{\gamma_R\left(\beta_R - \frac{\sqrt{V_1(t) \cdot V_2(t)}}{L_t}\right)}} \tag{13}$$

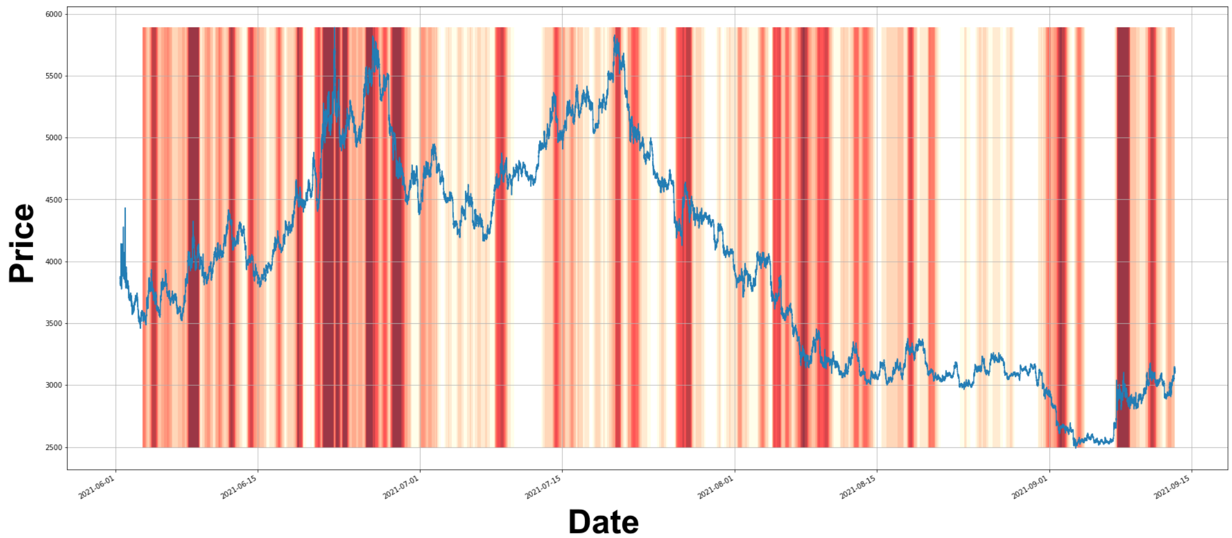
Dynamic Fee Usage

Based on the historical data of trades on Uniswap V3 pools, the adaptive fee values were calculated at different time points. The reaction of the fee function adequately corresponds to what is happening on the market.

On each chart, the white color corresponds to the zones with the lowest fees, and the dark red color corresponds to the highest.

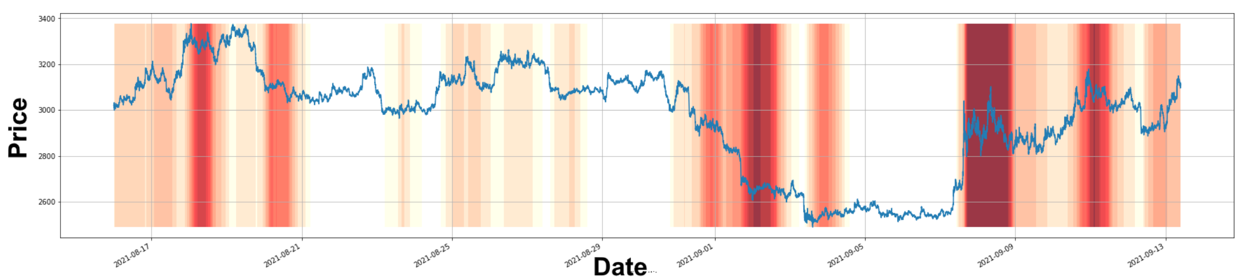
ETH/USDC

ETH/USDC price chart in a pool with a fee of 0.05%:

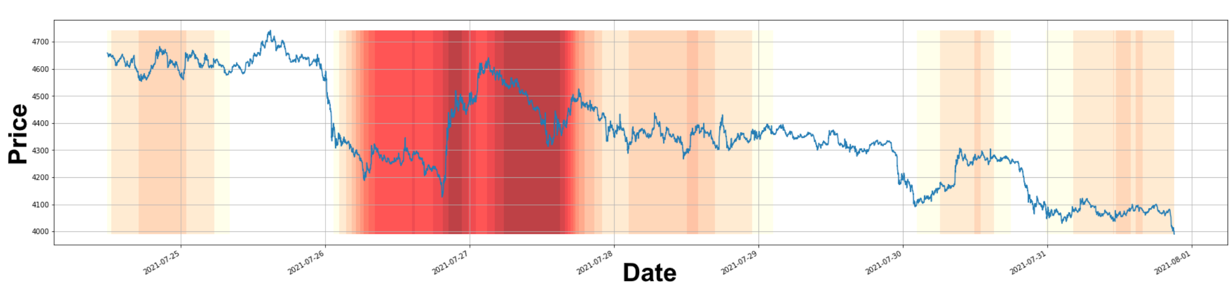


The graph above demonstrates that the control function successfully detects time intervals with high volatility and high demand for the asset, which indicates the need to increase the fee in dark red zones. Still, there are periods of relative calm with low trading volume and weak volatility, during which time it is reasonable to reduce the fee.

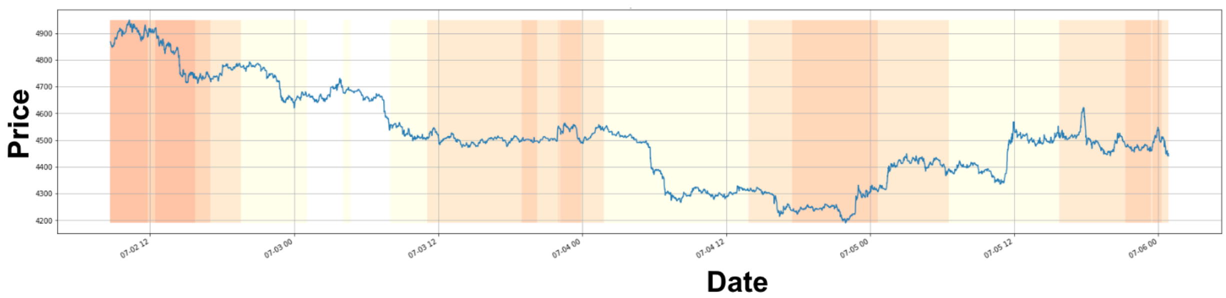
The more detailed period from 8/17/2021 to 9/13/2021:



The period from 7/25/2021 to 8/1/2021:



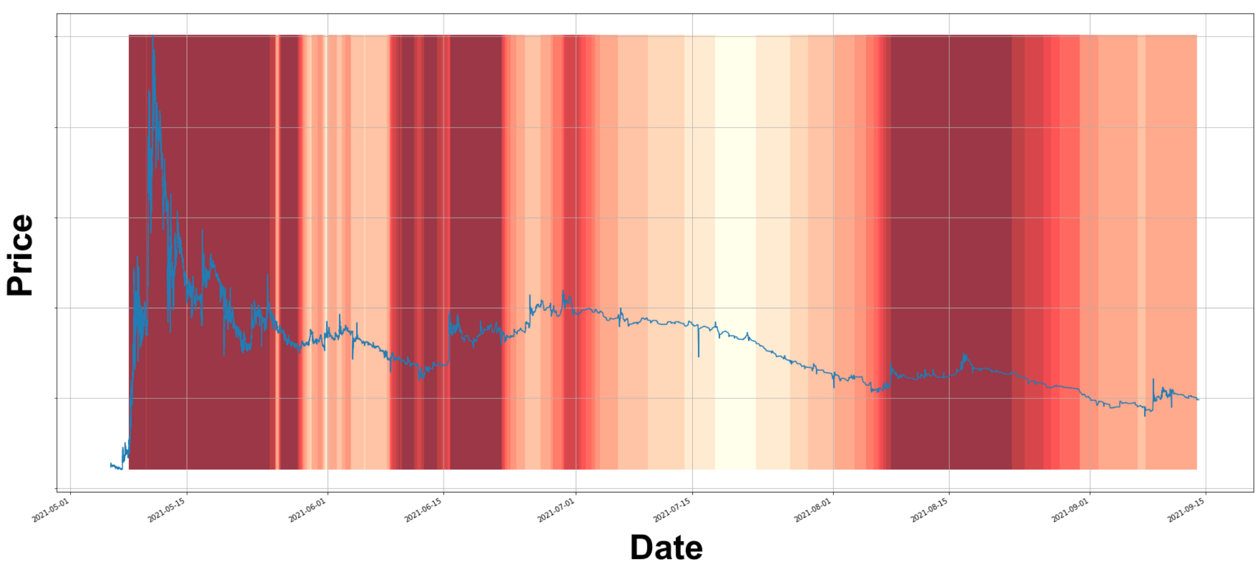
The period from 7/2/2021 to 7/6/2021:



In the more detailed charts, you can observe the fee's reaction to changes in market conditions in more detail. Thus, it is recommended to increase the fees drastically during moments where it collapses and rises, like in early September 2021, which simultaneously stimulates trading with a reduced commission during periods of calm.

SHIBA/ETH

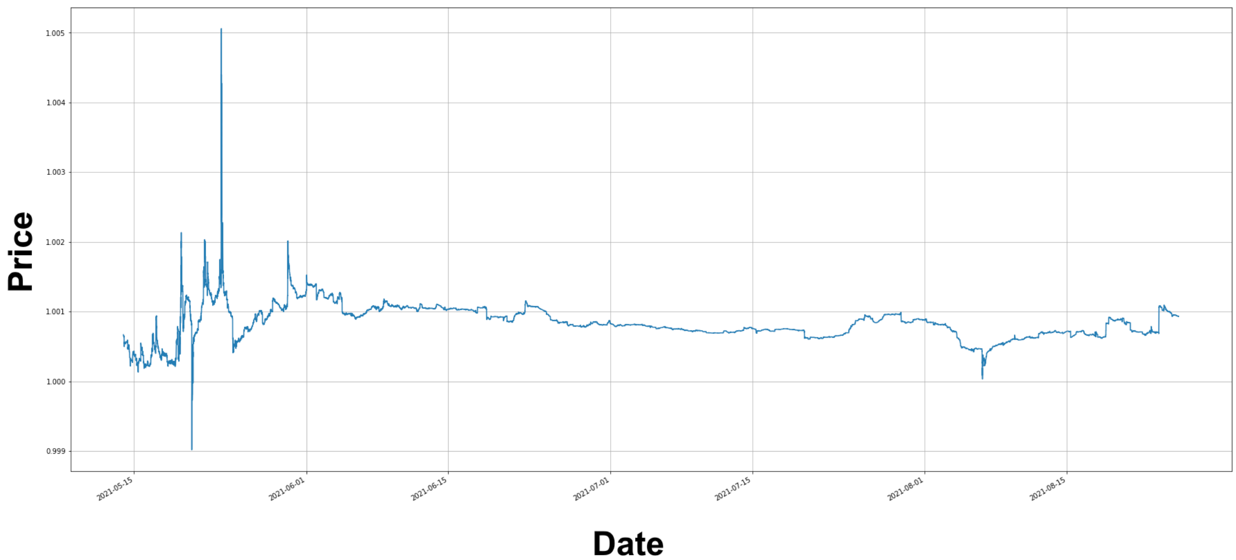
A similar graph of the same fee function for the SHIBA/ETH pool:



This chart reflects a pair with a more volatile and hyped asset, which forces the function to set a high fee most of the time.

DAI/USDC

A similar graph of the same fee function for the DAI/USDC pool:



A pool with a stablecoin has much lower volatility. This fact, combined with a stable trading volume, encourages the function to keep the fee at a minimum level.

Conclusion

According to the results of these tests, it can be concluded that the received fee function responds adequately and on time to changes in the nature of price behavior. A pair of stablecoins consistently has the lowest fees, and the fees of more volatile assets depend on the market situation. This is why such management of the commission will allow for balancing the interests of various traders, providing the most effective and favorable conditions.

2. Built-in Farming

Overview

After the user has provided liquidity for a certain range, he receives an ERC-721 token tied to this range. This token can be used, not only as a proof of the provided liquidity, but also as a tool for yield-farming.

To receive rewards, a liquidity provider can participate in one of the farming Campaigns. A Campaign is a time-limited, pool-based incentive program that contains a certain amount of reward tokens.

To participate in the Campaign, one needs to send an ERC-721 token to the farming contract, which will be stored on it until the end of the Campaign. After the end of the Campaign, all reward tokens will be distributed among the liquidity providers who participated in the Campaign, and all ERC-721 tokens will be returned back.

The amount of the reward that a particular liquidity provider will receive depends on the time while her liquidity was in the price range in which the trades took place.

For proper reward calculation for each Campaign participant we invented an entity called a **Virtual pool**. A **Virtual pool** is a simplified copy of a real pool; it is used to track the time while the liquidity of different providers is active.
The calculation of the time when liquidity was active for a certain period is performed as follows:

$$secondsPerLiquidity = \frac{rangeSecondsInside}{rangeLiquidity} \tag{15}$$

where *rangeSecondsInside* is the time while the price was at the range of provided liquidity, and *rangeLiquidity* is all of the liquidity on that range.

The time while liquidity was active on a given range is calculated as:

$$positionSecondsInside = secondsPerLiquidity * positionLiquidity \quad (16)$$

where *positionLiquidity* is the value that represents position's liquidity.

Now, knowing the time while the liquidity was active, we can calculate the reward for the given Campaign participant:

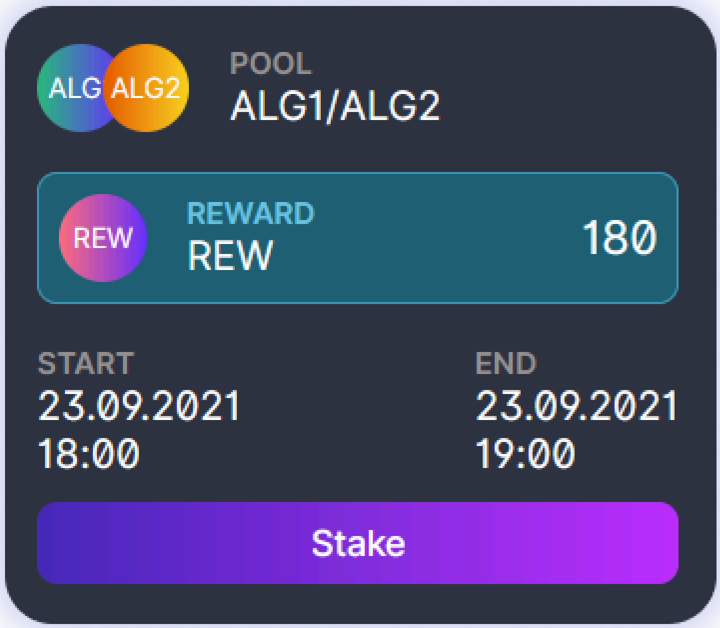
$$reward = \frac{totalReward * positionSecondsInside}{campaignDuration} \quad (17)$$

where *totalReward* is the total reward amount, and *campaignDuration* is the duration of the Campaign.

Every time there is a swap in the real pool, or a new participant takes part in the event, the Virtual pool recalculates the rewards for each participant of the Campaign.

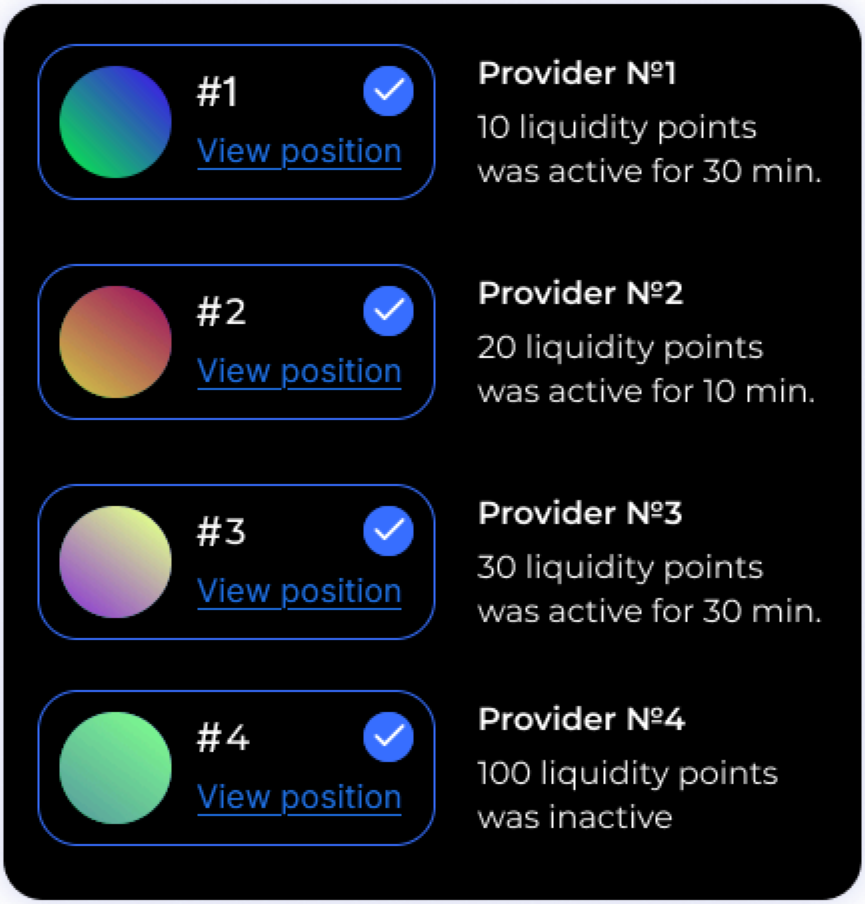
Example

Total reward amount of the Campaign is 180 tokens. The Campaign lasts 60 minutes.

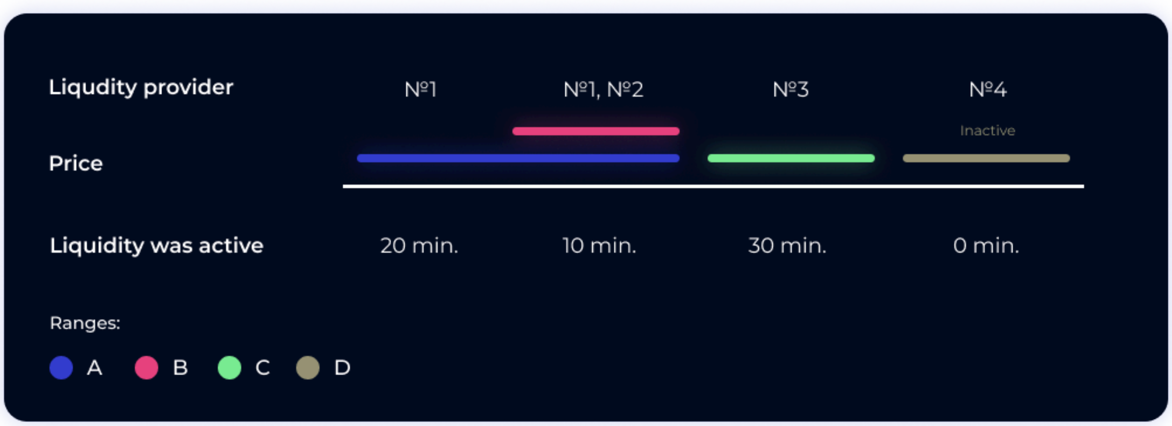


4 liquidity providers participate in the Campaign. Their positions have the following parameters:

Provider #1: Liquidity: 10 points, was active for 30 minutes
Provider #2: Liquidity: 20 points, was active for 10 minutes
Provider #3: Liquidity: 30 points, was active for 30 minutes
Provider #4: Liquidity: 100 points, was inactive



Let's depict the time while liquidity was active on ranges A, B, C, D:



It should be noted that the liquidity of Provider #4, located on range D, was inactive.

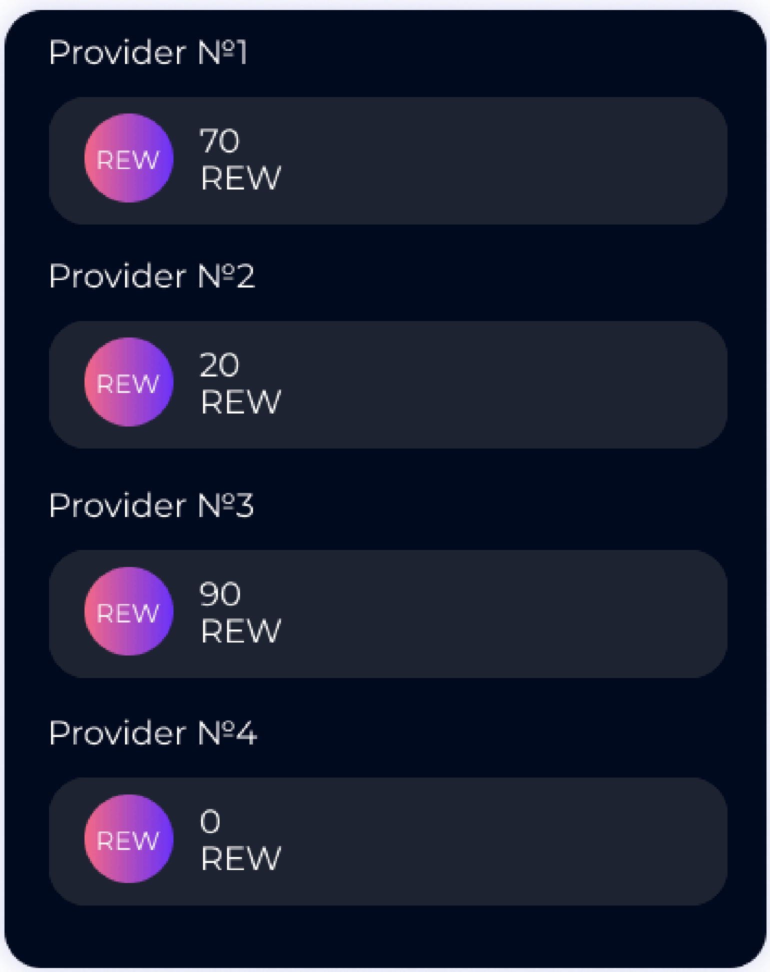
At the end of the Campaign, the rewards were distributed among the providers as follows:

Provider #1 received $(\frac{20*10}{10} + \frac{10*10}{30})/60 = \frac{7}{18}$ of total amount (70 tokens)

Provider #2 received $(\frac{10*20}{30})/60 = \frac{2}{18}$ of total amount (20 tokens)

Provider #3 received $(\frac{30*30}{30})/60 = \frac{9}{18}$ of total amount (90 tokens)

Despite the fact that 4 liquidity providers participated in the Campaign, the rewards were distributed only among those providers whose liquidity was active for at least some time. Provider #4 did not receive any awards.



Conclusion

Therefore, Algebra implements the following features for providing the best exchange experience to all the participants:

— An innovative adaptive fee formula guarantees optimal values for both liquidity providers and traders.

— Built-in farming allows projects to increase their attractiveness for holders and investors, in the meanwhile giving liquidity providers an opportunity to make profit from their deposited funds.

These features, as well as provide support for deflationary tokens, will place Algebra in a leading position in DeFi space.